AD-A050 565 MASSACHUSETTS INST OF TECH LEXINGTON LINCOLN LAB F/G 3/1
ON THE ACCURACY OF THE POSITIONS OF CELESTIAL OBJECTS DETERMINE--ETC(U)
DEC 77 L G TAFF F19628-78-C-0002
UNCLASSIFIED TN-1977-25 ESD-TR-77-343 NL

| OF |

AD A050565

END
DATE
FILMED
4 -78
DDC

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

ON THE ACCURACY OF THE POSITIONS OF CELESTIAL OBJECTS DETERMINED FROM A LINEAR MODEL

L. G. TAFF

Group 94

TECHNICAL NOTE 1977-25

22 DECEMBER 1977

LEXINGTON MASSACHUSETTS

ABSTRACT

This report discusses the positional accuracy obtainable from the local reduction of the topocentric coordinates of a celestial body in the topocentric reference frame. Both a theoretical and numerical analysis are provided. The method of dependences, the four constant plate model, and the six constant plate model are considered. The purely arithmetic results were obtained via Monte Carlo simulations.

TABLE OF CONTENTS

I. INTRODUCTION

This report discusses the systematic accuracy one can obtain when reducing the position of one celestial object (the program object) relative to other celestial objects (the reference objects). The heart of the problem is to delineate the number of reference objects needed and their arrangement with respect to the program object. The only other investigations in the astronomical literature appear to be the discussion by Plummer[1] for the method of dependences and the inquiry of Eichhorn and Williams[2] for the plate modeling procedure of classical photographic astrometry. Although neither of these is especially relevant to a real time reduction procedure, they do form the mathematical basis for the Precision Local Calibration procedure of the Ground-based Electro-Optical Deep Space Surveillance system (GEODSS). Neither simpler local calibration procedures nor global calibration procedures (such as the one used for the Anglo-Australian Telescope) will be discussed here. See Taff and Poirier[3] for an investigation of the former in the current context.

Historically, the method of dependences used with three reference objects (stars in fact although experimentation with galaxies is underway at the Lick Observatory) has dominated the subject. Some of the reasons for this were summarized in §IVB of Taff.[4] With the advent of automatic measuring machines, high speed, large storage, electronic digital computers, and the development of the plate overlap technique,[5] the situation has changed. Astrometers who use the latter tools prefer the maximum number of reference objects. More than a dozen per reduction is common. Since a large fraction of working astrometers still use the older procedures, a schism exists in the community. This report is my effort to contribute towards a rapprochement.

One could properly ask, "Why study the problems of classical photographic astrometry when designing a real time procedure where no photograph is ever taken, no plate ever measured, nor standard coordinates ever computed?" Until and unless the final configuration of the telescope/camera/video monitor combination is fixed (and thoroughly investigated with regard to accidental errors, systematic errors, and their couplings) the simplest, least presumptive local reduction procedure mimics (formally) the analyses of photographic astrometry. In addition, I expect to make a contribution to the astronomical literature complementing and supplementing the earlier work.[1, 2] Finally, by performing the statistical analysis correctly (neither Plummer[1] nor Eichhorn and Williams[2] did), by developing analytical predictions of the achievable accuracy, and by testing this analytical construct via extensive Monte Carlo simulations a more complete grasp of the problem can be at hand.

Following a brief section on notation, three different reduction procedures are explored. They are known as the method of dependences, the four constant plate model, and the six constant plate model. Numerical computations using both artificial reference objects and the stars of the Smithsonian Astrophysical Observatory Star Catalogue[6] are then presented for the six constant plate model. I adhere to the terminology of photographic astrometry but use a notation that minimizes the typist's difficulties. Before proceeding it is worth remarking that, when applicable, either global calibration or the simpler local calibration procedures will always use telescope time more economically than PLC does and will also produce results of comparable accuracy.

II. NOTATION

The constants entering the relationship between the measured coordinates and the real coordinates are called plate constants. The letters a-f are used for this purpose. The corresponding upper case letters symbolize their estimates obtained by a least squares analysis.

The reference objects lie on a plane and rectangular Cartesian coordinates are used. Thus (x_j, y_j) denotes the position of the j^{th} reference object. N is the total number of reference objects and, except for $\underline{R}$, all vectors are N-dimensional and all sums run from 1 to N. Thus,

$$\underline{x} = (x_1, x_2, \ldots, x_N),$$

$$\Sigma x_j = \sum_{j=1}^{N} x_j.$$

Furthermore, the coordinate system is always translated such that

$$\Sigma x_j = \Sigma y_j = 0.$$

Measured quantities are indicated by primes (as in y'). Upper case x and y denote the position of the program object and the vector $\underline{R} = (X, Y)$.

Finally, the moment of inertia tensor of the reference objects is symbolized by I (I^{-1} represents its inverse) and S is the area occupied by the reference objects.

III. THE METHOD OF DEPENDENCES

When a linear plate model is sufficiently precise to relate the measured values, $\underline{x}'$, $\underline{y}'$, to the true (in the connotation of real) values, $\underline{x}$, $\underline{y}$, a quantity called the dependence was introduced by Schlesinger.[7] When using N reference objects the vector of dependences is defined by

$$\underline{D} = \underline{i}/N + [X(\underline{x}I_{yy} - \underline{y}I_{xy}) + Y(\underline{y}I_{xx} - \underline{x}I_{xy})]/\det(I), \quad (1)$$

where $\underline{i}$ is an N-dimensional unit vector, (X, Y) are the true coordinates of the program object, and I is the moment of inertia tensor of the reference objects, viz,

$$I = \begin{pmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{pmatrix} = I^{\dagger}. \quad (2)$$

The quantity det(I) is the determinant of I. Clearly I is a non-negative semidefinite array. I can be singular if and only if the reference objects all lie on a straight line. This represents a degeneracy not further considered here. Hence, I is a positive definite array in the sequel.

The whole point of the method of dependences is the avoidance of the necessity of computing estimates for the plate constants in (a-f are the plate constants)

$$\underline{x} - \underline{x}' = a + b\underline{x} + c\underline{y}, \tag{3a}$$

$$\underline{y} - \underline{y}' = d + e\underline{x} + f\underline{y}. \tag{3b}$$

To see how this may be accomplished (using the dependences) observe that the constraints

$$\Sigma x_j = 0, \ \Sigma y_j = 0, \tag{4}$$

and the definition of I imply that

$$\underline{D} \bullet \underline{x} = X, \ \underline{D} \bullet \underline{y} = Y. \tag{5a}$$

From Eqs. (3) and the fact that [cf. Eqs. (1, 4)]

$$\Sigma D_j = 1,$$

it follows that

$$\underline{D} \bullet \underline{x}' = X', \ \underline{D} \bullet \underline{y}' = Y'. \tag{5b}$$

Whence, from Eqs. (5),

$$X = X' + \underline{D} \bullet (\underline{x} - \underline{x}'), \tag{6a}$$

$$Y = Y' + \underline{D} \bullet (\underline{y} - \underline{y}'), \tag{6b}$$

are equivalent to Eqs. (3). If the dependences did not depend on X or Y the problem would then be solved. In order to proceed assume the validity of the linear plate model. This implies a-f are small quantities. Then, to second-order,

$$\underline{D}(X, Y) \bullet (\underline{x} - \underline{x}') = \underline{D}(X', Y') \bullet (\underline{x} - \underline{x}')$$

and similarly for the y term. The use of this approximation in Eqs. (6) renders the computational problem explicit and a-f are not needed. The true power of this approximation is revealed when N = 3. Details are in Taff.[4] Hence, the tremendous hold, in a slide rule only era, of the N = 3 version of the method of dependences.

The final factor that established the dominance of this particular reduction technique lay in the error estimates for X, Y. In addition to producing essentially unbiased values for X and Y, the estimate of the variances, viz,

$$\begin{aligned} \mathrm{var}(X) &= (\partial X/\partial x')^2 \mathrm{var}(x') + \Sigma(\partial X/\partial x_j')^2 \mathrm{var}(x_j') \\ &= \sigma_x^2 (1 + \underline{D} \bullet \underline{D}), \end{aligned} \qquad (7a)$$

$$\mathrm{var}(Y) = \sigma_y^2 (1 + \underline{D} \bullet \underline{D}), \qquad (7b)$$

clinched the argument ($\sigma_{x,\ y}$ is the standard deviation of an individual measurement in the x, y direction).

From Eq. (1),

$$\underline{D} \bullet \underline{D} = 1/N + \underline{R} \bullet I^{-1} \bullet \underline{R}, \tag{7c}$$

where $\underline{R} = (X, Y)$ is the two-dimensional position vector of the program object and I^{-1} is the inverse of I. Since I is positive definite so is I^{-1}. Hence, the minimum variances for X and Y occur when the program object is placed at the center of mass of the reference objects and in this case

$$\mathrm{var}(X, Y) = \sigma^2_{x,\ y}(1 + 1/N). \tag{8}$$

The supporters of the N = 3 method of dependences reduction technique (see van de Kamp[8] for example) now argue that the increase in accuracy to be obtained by using more than three stars is at most $(4/3)^{1/2} - 1 = 15.5$ percent. Moreover, the relative gain in going from N = 3 to N = 4 is only $(16/15)^{1/2} - 1 = 3.3$ percent. This argument is false because Eq. (8) only applies (ignoring the sleight of hand used to derive it) for N > 3. See §V below.

Notwithstanding the fact that the result embodied in Eqs. (7) has been arrived at heuristically, I shall reproduce it with full rigor for the four constant plate model and approximately for the six constant plate model. Moreover, its essential predictions are confirmed by the Monte Carlo simulations described in §VI. Hence, a further discussion is presented here. By examining Eqs. (7) one concludes that (for N above the minimum!)

(i) The optimum position for the program object is at $R = 0$,

(ii) When $R = 0$ the coordinate variances scale linearly with the measuring variance,

(iii) When $R = 0$ increasing the number of reference stars beyond the minimum offers little improvement in accuracy (e.g., it's 2.1 percent for $N = 4 \rightarrow N = 5$),

(iv) When $R = 0$ the expected accuracy is independent of the areal extent of the reference objects,

(v) When $R \neq 0$,

$$\mathrm{var}(X, Y) = \sigma^2_{x,\,y}[1 + (1 + kR^2/S)/N], \qquad (9)$$

where S is the areal extent of the reference stars and k is a constant $\simeq 10$ (it depends on the radii of gyration). Thus, if $R \neq 0$, one wants the maximum areal extent so that the full x and y ranges can be used to accurately determine the plate constants. The derivation of Eq. (9) is postponed to §IV.

IV. THE FOUR CONSTANT PLATE MODEL

In certain situations a similarity transform suffices to relate the measured coordinates to the true coordinates. When this is so an orientation angle, θ, a scale factor, ρ, and two zero point constants, c, d, uniquely determine the model. In particular,

$$\underline{x} - \underline{x}' = \rho(\underline{x}\cos\theta + \underline{y}\sin\theta) + c,$$
$$\underline{y} - \underline{y}' = \rho(-\underline{x}\sin\theta + \underline{y}\cos\theta) + d,$$

or

$$\underline{x} - \underline{x}' = a\underline{x} + b\underline{y} + c,$$
$$\underline{y} - \underline{y}' = -b\underline{x} + a\underline{y} + d.$$

Note that the plate constants do not have the same meaning as in Eqs. (3). The least squares solution for the estimators of a-d is both unique and trivial (since the matrix of the normal equations is diagonal). From the covariance matrix the only non-zero terms are ($\sigma_x = \sigma_y = \sigma$ necessarily)

$$\text{var}(A) = \text{var}(B) = \sigma^2/\text{Tr}(I),$$
$$\text{var}(C) = \text{var}(D) = \sigma^2/N.$$

The variance of X or Y, which are obtained by solving

$$X - X' = AX + BY + C,$$

$$Y - Y' = -BX + AY + D,$$

is given by

$$\mathrm{var}(X, Y) = \sigma^2[1 + 1/N + R^2/\mathrm{Tr}(I)]. \quad (10)$$

If $R \neq 0$, since $\mathrm{Tr}(I) = 2\det^{1/2}(I)$, and

$$\det^{1/2}(I) = I_{xx} = \underline{x} \bullet \underline{x} = \kappa^2 N \propto SN,$$

where κ is the radius of gyration, Eq. (10) can be transformed into

$$\mathrm{var}(X, Y) = \sigma^2[1 + (1 + kR^2/S)/N]. \quad (11)$$

(For a uniform distribution of reference objects $k = 12$ while for a distribution on the perimeter of a square with sides of length $S^{1/2}$, $k = 4$.) Hence, Eqs. (7) are rigorously derived. It is also important to note that the only assumption made is that the noise associated with a measurement has a constant mean (not necessarily zero) and is uncorrelated.

The four constant model is so special we cannot expect it to apply to the problem of GEODSS. I considered here because its symmetry allows a complete solution to the problem which can be used as a guide in the more complex case.

V. THE SIX CONSTANT PLATE MODEL

The plate model is given in Eqs. (3) and represents an affine transformation. From the normal equations we can compute the covariance matrix, and only

$$\mathrm{var}(A, D) = \sigma^2_{x,\, y}/N,$$

$$\mathrm{var}(B, E) = \sigma^2_{x,\, y} I_{yy}/\det(I),$$

$$\mathrm{var}(C, F) = \sigma^2_{x,\, y} I_{xx}/\det(I),$$

$$\mathrm{cov}(B, C) = -\sigma^2_x I_{xy}/\det(I),$$

$$\mathrm{cov}(E, F) = -\sigma^2_y I_{xy}/\det(I),$$

are non-zero. The least squares solution provides unbiased estimators for X and Y. The unbiased estimators for $\sigma_{x,\, y}$ are

$$s_{x,\, y} = (x,\, y \text{ residual})/(N - 3),\ N > 3.$$

Clearly, if N = 3 there is no statistical problem at all and both the x and y residuals vanish. This is why Eq. (8) cannot be used to deduce the relative precision of the N = 3, N > 3 cases.

The coordinates of the program object are obtained by inverting

$$X - X' = AX + BY + C, \tag{12a}$$

$$Y - Y' = DX + EY + F, \tag{12b}$$

which has a determinant of

$$d = (1 - B)(1 - F) - EC.$$

The variances of X and Y are given by

$$\mathrm{var}(X) = [\sigma_x^2(1 - F)^2 + \sigma_y^2 C^2][1 + 1/N + \underline{R} \bullet I^{-1} \bullet \underline{R}]/d^2, \tag{13a}$$

$$\mathrm{var}(Y) = [\sigma_y^2(1 - B)^2 + \sigma_x^2 E^2][1 + 1/N + \underline{R} \bullet I^{-1} \bullet \underline{R}]/d^2. \tag{13b}$$

The four constant plate model has sufficient symmetry that the terms appearing in Eqs. (13) cancelled in Eq. (10).

Further rigorous progress appears to be impossible. However, after averaging var(X, Y) over the noise one is led to

$$\langle \mathrm{var}(X, Y) \rangle \simeq \sigma_{x,\, y}^2 [1 + 1/N + \underline{R} \bullet I^{-1} \bullet \underline{R}], \tag{13c}$$

exactly as before. Also, as in §IV, the $\underline{R} \bullet I^{-1} \bullet \underline{R}$ term can be shown to be proportional to kR^2/NS.

VI. MONTE CARLO SIMULATIONS

For the six constant model Eqs. (13) provides all of the information we can obtain concerning the expected accuracy of the results. This is true if and only if $N > 3$. Hence the need of a simulation to elucidate the difference between the $N = 3$ results and the $N > 3$ results. Moreover, if we assume a uniform distribution of reference objects is optimal [a result implicit in Eqs. (13c)], the Monte Carlo technique will truly mimic the real problem. However, neither the distribution of stars on the celestial sphere nor the distribution of stars in catalogues is uniform. Thus, the SAOC was also used to provide the positions of the reference objects.

A. Simulations Using The Uniform Distribution

To further explore the implications of Eqs. (13) a total of 360 different simulations predicated on three different program object locations relative to the reference objects were performed. Each individual result represents the average of 10,000 trials. Hence, the expected accuracy is one percent. The values of N used were 3(1)8, in each case; $\sigma_x = \sigma_y = \sigma$ and σ was $1''.25$, $2''.50$, $5''.00$, or $10''.00$; and in each case $L_x = L_y = L = S^{1/2}$ with $L = 0^\circ.25$, $0^\circ.50$, $1^\circ.00$, $2^\circ.00$, or $4^\circ.00$. The smaller two values of L bracket the density of the SAOC, the largest two values of L correspond to the density of the FK5(= FK4 plus the FK4 Sup), and the intermediate value of L is appropriate for the AGK2A plus the SRS. The largest value of σ reflects the current configuration of the Experimental Test Site of the GEODSS system. The smallest value of σ represents that expected in the deployed GEODSS system. The intermediate values refer to making improvements in the telescope, or the telescope and video monitor.

The three different program object locations will be referred to as the $R = 0$, the min(R), and the max(R) situations:

(i) $R = 0$; the program object was always placed exactly at the center of mass of the N reference stars,

(ii) *min(R); N + 1 positions were generated* and the one closest to the origin was chosen to be the program object (the real case),

(iii) max(R); the program object was placed somewhere within the smallest rectangle containing all of the reference objects (primarily to explore the kR^2/NS term).

Since it is impossible to theoretically construct error estimates for X and Y in the $N = 3$ case, all of the analysis used the absolute deviations, viz,

$$\Delta_x = |X - X^*|, \; \Delta_y = |Y - Y^*|, \tag{14a}$$

$$\Delta^2 = \Delta_x^2 + \Delta_y^2, \tag{14b}$$

where (X, Y) is the true position of the program object and (X^*, Y^*) is the position obtained by inverting the model [cf. Eqs. (12)]. $\Delta_{x,\,y}$ should have the same functional dependence on $\sigma_{x,\,y}$, N, and kR^2/NS as var(X, Y) do. The results are too voluminous to reproduce here. Tables 1, 2, and 3 summarize some of them. As the x and y dimensions were treated similarly the properly weighted average of Δ_x and Δ_y only is reported (in the $\Delta_{x,\,y}/\sigma$ columns).

A careful perusal of these tables verifies the predictions on page 10. Moreover, while increasing N beyond five is not warranted, the increase from N = 3 to N = 4 results in average improvement of 2.6 in the position. This of course, is still not the entire story. For as much as I know both (X, Y) and (X^*, Y^*) the higher moments of $\Delta_{x,\,y}$ and Δ can be computed. The ratios of the N = 3 to N = 4, 5 and the N = 4 to N = 5 standard deviations for the min(R) case (L = $1^\circ.00$) are given in Table 4. The enormous reduction in going from N = 3 to N = 4 precludes any thoughts of using only three reference objects and the six plate constant model.

B. Simulations Using The SAOC

To obtain a much more accurate feel for the quality of the results when the reference object distribution is

TABLE 1

R = 0 RESULTS

	$\Delta_{x,y}/\sigma$					Δ/σ			
				L = $0\fdg25$					
N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$	N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$
3	0.947	0.971	1.03	1.12	3	1.49	1.52	1.70	1.76
4	0.896	0.897	0.898	0.902	4	1.41	1.41	1.41	1.42
5	0.874	0.874	0.874	0.875	5	1.37	1.37	1.37	1.37
6	0.860	0.860	0.860	0.861	6	1.35	1.35	1.35	1.35
7	0.861	0.861	0.861	0.861	7	1.35	1.35	1.35	1.35
8	0.851	0.851	0.851	0.851	8	1.33	1.33	1.33	1.33
				L = $0\fdg50$					
N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$	N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$
3	0.937	0.947	0.971	1.03	3	1.47	1.49	1.52	1.70
4	0.895	0.896	0.897	0.898	4	1.41	1.41	1.41	1.41
5	0.874	0.874	0.877	0.874	5	1.37	1.37	1.37	1.37
6	0.860	0.860	0.860	0.860	6	1.35	1.35	1.35	1.35
7	0.861	0.861	0.861	0.861	7	1.35	1.35	1.35	1.35
8	0.851	0.851	0.851	0.851	8	1.33	1.33	1.33	1.33
				L = $1\fdg00$					
N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$	N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$
3	0.944	0.937	0.547	0.971	3	1.49	1.47	1.49	1.52
4	0.895	0.895	0.896	0.897	4	1.41	1.41	1.41	1.41
5	0.874	0.874	0.874	0.874	5	1.37	1.37	1.37	1.37
6	0.860	0.860	0.860	0.860	6	1.35	1.35	1.35	1.35
7	0.861	0.861	0.861	0.861	7	1.35	1.35	1.35	1.35
8	0.851	0.851	0.851	0.851	8	1.33	1.33	1.33	1.33
				L = $2\fdg00$					
N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$	N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$
3	0.930	0.944	0.937	0.947	3	1.46	1.49	1.47	1.49
4	0.895	0.895	0.895	0.896	4	1.41	1.41	1.41	1.41
5	0.874	0.874	0.874	0.874	5	1.37	1.37	1.37	1.37
6	0.860	0.860	0.860	0.860	6	1.35	1.35	1.35	1.35
7	0.861	0.861	0.861	0.861	7	1.35	1.35	1.35	1.35
8	0.851	0.851	0.851	0.851	8	1.33	1.33	1.33	1.33
				L = $4\fdg00$					
N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$	N/σ	$1\farcs25$	$2\farcs50$	$5\farcs00$	$10\farcs00$
3	0.930	0.929	0.944	0.937	3	1.46	1.46	1.49	1.47
4	0.895	0.895	0.895	0.895	4	1.41	1.41	1.41	1.41
5	0.874	0.874	0.874	0.874	5	1.37	1.37	1.37	1.37
6	0.860	0.860	0.860	0.860	6	1.35	1.35	1.35	1.35
7	0.861	0.861	0.861	0.861	7	1.35	1.35	1.35	1.35
8	0.851	0.851	0.851	0.851	8	1.33	1.33	1.33	1.33

TABLE 2

MIN(R) RESULTS

	$\Delta_{x,y}/\sigma$					Δ/σ			
				L = 0°25					
N/σ	1″25	2″50	5″00	10″00	N/σ	1″25	2″50	5″00	10″00
3	3.09	2.17	1.95	2.17	3	5.01	3.54	3.10	3.60
4	1.08	1.08	1.08	1.09	4	1.69	1.70	1.71	1.73
5	0.947	0.947	0.947	0.948	5	1.49	1.49	1.49	1.49
6	0.908	0.908	0.908	0.908	6	1.43	1.43	1.43	1.43
7	0.885	0.884	0.884	0.885	7	1.39	1.39	1.39	1.39
8	0.882	0.882	0.882	0.882	8	1.38	1.38	1.38	1.38
				L = 0°50					
N/σ	1″25	2″50	5″00	10″00	N/σ	1″25	2″50	5″00	10″00
3	2.29	3.09	2.17	1.95	3	3.82	5.01	3.54	3.10
4	1.08	1.08	1.08	1.08	4	1.69	1.69	1.70	1.71
5	0.947	0.947	0.947	0.947	5	1.49	1.49	1.49	1.49
6	0.908	0.908	0.908	0.908	6	1.43	1.43	1.43	1.43
7	0.885	0.885	0.884	0.884	7	1.39	1.39	1.39	1.39
8	0.882	0.882	0.882	0.882	8	1.38	1.38	1.38	1.38
				L = 1°00					
N/σ	1″25	2″50	5″00	10″00	N/σ	1″25	2″50	5″00	10″00
3	2.07	2.29	3.09	2.17	3	3.32	3.82	5.01	3.54
4	1.08	1.08	1.08	1.08	4	1.69	1.69	1.69	1.70
5	0.947	0.947	0.947	0.947	5	1.49	1.49	1.49	1.49
6	0.908	0.908	0.908	0.908	6	1.43	1.43	1.43	1.43
7	0.885	0.885	0.884	0.884	7	1.39	1.39	1.39	1.39
8	0.882	0.882	0.882	0.882	8	1.38	1.38	1.38	1.38
				L = 2°00					
N/σ	1″25	2″50	5″00	10″00	N/σ	1″25	2″50	5″00	10″00
3	2.14	2.07	2.29	3.09	3	3.44	3.32	3.82	5.01
4	1.08	1.08	1.08	1.08	4	1.69	1.69	1.69	1.69
5	0.947	0.947	0.947	0.947	5	1.49	1.49	1.49	1.49
6	0.908	0.908	0.908	0.908	6	1.43	1.43	1.43	1.43
7	0.885	0.885	0.884	0.884	7	1.39	1.39	1.39	1.39
8	0.882	0.882	0.882	0.882	8	1.38	1.38	1.38	1.38
				L = 4°00					
N/σ	1″25	2″50	5″00	10″00	N/σ	1″25	2″50	5″00	10″00
3	9.16	2.14	2.07	2.29	3	16.2	3.44	3.32	3.82
4	1.08	1.08	1.08	1.08	4	1.69	1.69	1.69	1.69
5	0.947	0.947	0.947	0.947	5	1.49	1.49	1.49	1.49
6	0.908	0.908	0.908	0.908	6	1.43	1.43	1.43	1.43
7	0.885	0.885	0.884	0.884	7	1.39	1.39	1.39	1.39
8	0.882	0.882	0.882	0.882	8	1.38	1.38	1.38	1.38

TABLE 3

MAX(R) RESULTS

L = 0°25

$\Delta_{x,y}/\sigma$					Δ/σ				
N/σ	1''25	2''50	5''00	10''00	N/σ	1''25	2''50	5''00	10''00
3	30.5	2.36	2.82	30.2	3	64.7	3.73	4.42	58.9
4	1.14	1.14	1.14	1.16	4	1.79	1.79	1.79	1.81
5	1.02	1.02	1.02	1.03	5	1.61	1.61	1.61	1.61
6	0.982	0.982	0.982	0.983	6	1.54	1.54	1.54	1.54
7	0.948	0.948	0.948	0.949	7	1.49	1.49	1.49	1.49
8	0.938	0.938	0.938	0.939	8	1.47	1.47	1.47	1.47

L = 0°50

N/σ	1''25	2''50	5''00	10''00	N/σ	1''25	2''50	5''00	10''00
3	2.66	3.47	3.10	2.27	3	4.26	8.81	5.83	3.65
4	1.15	1.15	1.15	1.14	4	1.80	1.80	1.79	1.79
5	1.02	1.02	1.02	1.02	5	1.61	1.61	1.61	1.61
6	0.982	0.982	0.982	0.982	6	1.54	1.54	1.54	1.54
7	0.948	0.948	0.948	0.948	7	1.49	1.49	1.49	1.49
8	0.938	0.938	0.938	0.938	8	1.47	1.47	1.47	1.47

TABLE 4

RATIOS OF STANDARD DEVIATIONS ($L = 1^\circ.00$)

σ	Δ_3/Δ_4	Δ_3/Δ_5	Δ_4/Δ_5
$1''.25$	14.6	25.0	1.72
2.50	33.7	57.3	1.70
5.00	59.0	102	1.73
10.00	16.5	34.1	2.06

non-uniform the SAOC was used to supply positions for the reference objects. Only a $0^{\circ}\!.75$ radius field was used but the same values of N and σ as above were tried. The right ascension and declination of the field centers were chosen to be $\alpha = 0^h(0^h\!.5)23^h\!.5$, $\delta = -85^{\circ}(5^{\circ})85^{\circ}$. Table 5 presents the all-sky averages for the sundry values of N and σ. The simple functional dependences noted above are still present as is the slow decrease in accuracy beyond N = 5. The variance of Δ again plays an important role in directing our attention to the inadequacy of N = 3.

A closer examination of the individual results shows that N = 3 can yield values of Δ (for $\sigma = 2''\!.50$) in excess of 200". Presumably this is due to a near collinear alignment of the reference objects in this case (i.e., I is nearly singular).

TABLE 5

SAOC RESULTS

	σ	Δ	$\mathrm{var}^{1/2}(\Delta)$		σ	Δ	$\mathrm{var}^{1/2}(\Delta)$
N = 3	1"25	5"4	30"4	N = 6	1"25	1"42	0"98
	2.50	81.7	2634		2.50	2.96	1.94
	5.00	28.3	265.6		5.00	5.63	3.78
	10.00	54.6	455.7		10.00	12.0	8.0
N = 4	1.25	1.94	2.70	N = 7	1.25	1.39	1.00
	2.50	3.90	4.51		2.50	2.79	1.98
	5.00	7.67	10.14		5.00	5.42	3.84
	10.00	15.2	16.0		10.00	11.0	7.7
N = 5	1.25	1.56	1.04	N = 8	1.25	1.31	1.00
	2.50	3.07	1.98		2.50	2.52	1.96
	5.00	6.06	4.20		5.00	5.31	4.02
	10.00	12.6	8.2		10.00	10.5	8.1

ACKNOWLEDGMENT

I wish to express my gratitude to Iva M. Poirier for providing both the necessary stimulus and the assistance needed to complete this work.

REFERENCES

1. H. C. Plummer, Mon. Not. R. Astr. Soc. 92, 892 (1932).

2. H. Eichhorn and C. A. Williams, Astron. J. 68, 221 (1963).

3. L. G. Taff and I. M. Poirier, "The Current State of GEODSS Astrometry," Project Report, Lincoln Laboratory, M.I.T. (to be published).

4. L. G. Taff, "Astrometry In Small Fields," Technical Note 1977-2, Lincoln Laboratory, M.I.T. (14 June 1977), DDC AD-A043568.

5. H. Eichhorn, Astron. Nach. 285, 233 (1960).

6. Smithsonian Astrophysical Observatory (Staff of), Star Catalogue, Smithsonian Institute No. 4562, Wash. DC, (1966).

7. F. Schlesinger, Astrophys. J. 33, 161 (1911).

8. P. van de Kamp, Principles of Astrometry (W. H. Freeman and Co., London, 1967).

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

18 19

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
ESD-TR-77-343

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6
On the Accuracy of the Positions of Celestial Objects Determined from a Linear Model.

5. TYPE OF REPORT & PERIOD COVERED
9
Technical Note

6. PERFORMING ORG. REPORT NUMBER
Technical Note 1977-25

7. AUTHOR(s)
10
Laurence G. Taff

8. CONTRACT OR GRANT NUMBER(s)
15
F19628-78-C-0002

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Lincoln Laboratory, M.I.T.
P.O. Box 73
Lexington, MA 02173

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
16
Program Element No. 63428F
Project No. 2128

11. CONTROLLING OFFICE NAME AND ADDRESS
Air Force Systems Command, USAF
Andrews AFB
Washington, DC 20331

12. REPORT DATE
11
22 December 1977

13. NUMBER OF PAGES
34

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
Electronic Systems Division
Hanscom AFB
Bedford, MA 01731

12 32p.

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

14 TN-1977-25

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

celestial body	plate model	SAOC
topocentric	Monte Carlo simulations	GEODSS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report discusses the positional accuracy obtainable from the local reduction of the topocentric coordinates of a celestial body in the topocentric reference frame. Both a theoretical and numerical analysis are provided. The method of dependences, the four constant plate model, and the six constant plate model are considered. The purely arithmetic results were obtained via Monte Carlo simulations.

207650

LB